Denkraum: Degree of belief in a proposition H

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Contents

1 Probability and Bayes

Probability

- Long-term frequency
- Degree of belief

From Prior to Posterior probability

- Adjusting the "prior probability" of H, $\Pr(H)$
- by empirical data D [1].
- using the "marginal likelihood of the data" $\Pr(D)$ [2].
- leads to "posterior probability" of H, $\Pr(H \mid D)$
- in general, we reallocate *credibility* across the *space of candidate possibilities*.

Bayes theorem

- Named after Thomas Bayes (1702-1761): "An essay in towards solving a problem in the doctrine of chances" (1763) ("Versuch zur Lösung eines Problems der Wahrscheinlichkeitsrechnung")
- Bayesian Statistics: Standard in the 18th/19th century
- 20th century: Classical/Frequentist Statistics, more "objective"
- Bayesian statistics impractical until recently due to computational limitations

$$\boxed{\Pr(H \mid D) = \frac{\Pr(D \mid H) \times \Pr(H)}{\Pr(D)}} \tag{1}$$

[1] the "model" or the "likelihood" for data D is, $\Pr(D \mid H)$ (only this is used in *classical* statistics).

[2] $\Pr(D) = \Pr(D \mid H)\Pr(H) + \Pr(D \mid \textit{not}\, H)\Pr(\textit{not}\, H)$.

2 Null hypothesis significance testing (NHST)

- p-values, $p < 0.05$, etc.
- Scientific papers report findings that are "statistically significant", that is $p < 0.05$
- and then often treat those results as evidence of the alternative hypothesis in the discussion
- but the only thing we have is the rarity of a test statistic T (direct algebraic function of the data values)

  $$\boxed{p = \Pr(T \geq t_{observed} \mid H_0)} \tag{2}$$

  (2) the p-value only quantifies the *rarity* of the data relative to the *space of all possible data that might have been observed from the intended sampling process if the null hypothesis* H_0 *were true.*
- Often H_0: "no effect", "no correlation" etc., that is, theories are confirmed by "any" increase and increased precision leads to decreased corroborability of our theories
- whereas in "good" science, precision increases corroborabilty of our theories
- What we want are quantities of the form $\Pr(H \mid D)$.

3 Simple example: Estimation of an unknown mean

Data: $\bar{x} = 125, s = 10, n = 25$ ($SE_{\bar{X}} = 5$).

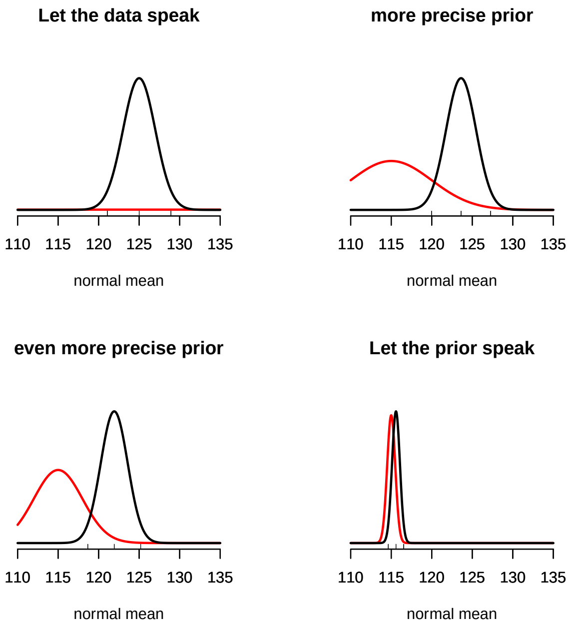

Figure 1: Prior (red curves) and Posterior distributions (black) for μ for the same data ($\bar{x} = 125, s = 10, n = 25$) with different priors, $\mu_0 = 115, \sigma_0 = \infty, 5, 3, 0.5$